

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Question Set 5

1 (a) Find $\frac{d}{dx}\left(x^3 - 3x + \frac{5}{x^2}\right) \cdot 3x^2 - 3 - \frac{10}{x^3}$ [3]

(b) Find $\int\left(6x^2 - \frac{2}{x^3}\right)dx$. $2x^3 + \frac{1}{x^2}$ [3]

2 Points A and B have position vectors $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ respectively.

Point C has position vector $\begin{pmatrix} p \\ 1 \end{pmatrix}$ and ABC is a straight line. $\vec{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

(a) Find p . $\vec{BC} = k\vec{AB}$ [2]

Point D has position vector $\begin{pmatrix} q \\ 1 \end{pmatrix}$ and angle $ABD = 90^\circ$. $\begin{pmatrix} p-1 \\ -1 \end{pmatrix} = k\begin{pmatrix} 4 \\ -2 \end{pmatrix} \quad k=0.5 \quad p=3$

(b) Determine the value of q . $\vec{BD} = \begin{pmatrix} q-1 \\ 1 \end{pmatrix}$ [3]

3 In this question you must show detailed reasoning. $\vec{BD} \cdot \vec{AB} = 0 \quad \begin{pmatrix} q-1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -2 \end{pmatrix} = 0 \quad 4q-4+2=0 \quad 4q-2=0 \quad q=0.5$

(a) Solve the equation $4\sin^2\theta = \tan^2\theta$ for $0^\circ \leq \theta \leq 180^\circ$. $4 = \frac{1}{\cos^2\theta} \quad \cos^2\theta = \pm 0.5 \quad \theta = 60, 120$ [5]

(b) Prove that $\frac{\sin^2\theta - 1 + \cos\theta}{1 - \cos\theta} \equiv \cos\theta \quad (\cos\theta \neq 1)$. $\frac{(-\cos^2\theta) - 1 + \cos\theta}{1 - \cos\theta} = \frac{\cos\theta - \cos^2\theta}{1 - \cos\theta} = \frac{\cos\theta(1 - \cos\theta)}{(1 - \cos\theta)} = \cos\theta$ [3]

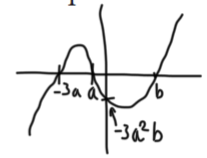
4 (a) Expand $(1+x)^4$. $x^4 + 4x^3 + 6x^2 + 4x + 1$ [1]

(b) Use your expansion to determine the exact value of 1002^4 . $\text{let } x=1001 \quad (1001)^4 + 4(1001)^3 + 6(1001)^2 + 4(1001) + 1 = 1.008$ [4]

5 The function f is defined by $f(x) = (x+a)(x+3a)(x-b)$ where a and b are positive integers.

(a) On the axes in the Printed Answer Booklet, sketch the curve $y = f(x)$. [2]

(b) On your sketch show, in terms of a and b , the coordinates of the points where the curve meets the axes. [2]



It is now given that $a = 1$ and $b = 4$.

(c) Find the total area enclosed between the curve $y = f(x)$ and the x -axis. $\int_{-3}^{-1} f(x)dx - \int_{-1}^4 f(x)dx = 8 + \frac{375}{4} = 101.75$ [4]

6 In this question you must show detailed reasoning.

(a) Solve the inequality $x^2 + x - 6 > 0$, giving your answer in set notation. $(x+3)(x-2) > 0 \quad x > 2 \text{ or } x < -3 \quad x: x(x > 2 \cup x < -3)$ [4]

(b) Solve the equation $x^3 - 7x^{\frac{3}{2}} - 8 = 0$. $\text{let } y^2 = x^{\frac{3}{2}} \quad y^6 - 7y^3 - 8 = 0 \quad (y^3 - 8)(y^3 + 1) \quad y = 2 \quad x = \sqrt[3]{4} \text{ only } \sqrt[3]{4} \text{ is a solution}$ [4]

(c) Find the exact solution of the equation $(3^x)^2 = 3 \times 2^x$. $(3^x)^2 = 3 \times 2^x \quad x(2 \log 3 - \log 2) = \log 3$
 $\log(3^x)^2 = \log 3 + \log 2^x \quad x = \frac{\log 3}{2 \log 3 - \log 2}$
 $2x \log 3 = \log 3 + x \log 2 \quad x = \frac{\log 3}{2 \log 3 - \log 2}$ [5]

7 Determine the points of intersection of the curve $3xy + x^2 + 14 = 0$ and the line $x + 2y = 4$. [5]

$$3x\left(-\frac{x}{2} + 2\right) + x^2 + 14 = 0$$

$$y = -\frac{x}{2} + 2$$

$$-\frac{3x^2}{2} + 6x + x^2 + 14 = 0$$

$$-3x^2 + 12x + 2x^2 + 28 = 0$$

$$x^2 - 12x - 28 = 0 \quad x = -2 \quad y = 3 \quad (-2, 3)$$

$$(x+2)(x-14) \quad x = 14 \quad y = -5 \quad (14, -5)$$

Total Marks for Question Set 5: 50

OCR

Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge